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Backscatter from a Moving Target in a Randomly Fluctuating Slab (Application to a Vegetated Environment)

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BACKSCATTER FROM A MOVING TARGET IN A RANDOMLY FLUCTUATING SLAB (APPLICATION TO A VEGETATED ENVIRONMENT)

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ABSTRACT

The stochastic nature of the target return depends on numerous factors: the spatial randomness of the forest's EM parameters, their temporal fluctuations, the target size and its state of motion. Our attention is focused upon the calculation of the mean power and the temporal spectrum of the target return, quantities of direct and obvious relevance. The spectral modifications are traceable to two first order factors: (1) the motion of the scattering centers within the random vegetation slab and (2) the target motion through a randomly inhomogeneous field, with the latter likely to dominate.

The analysis is carried out within the framework of the so-called "distorted wave Born approximation." The incoherent (random) scattering is accounted for to the order of single scatter, while coherent (background) effects such as refraction at the air-vegetation interface and ground reflections are properly considered. The rather laborious analytical sequence leads (via a set of carefully listed simplifying assumptions) to final results which are analytically simple, numerically tractable and their physical content readily interpretable. Two major assumptions concerning target characteristics should be mentioned: (1) the study dealt exclusively with point targets and (2) the target is presumed to move toward or away from the radar with constant velocity.

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CONTENTS

A. INTRODUCTION

The stochastic nature of the target return depends on numerous factors, the spatial randomness of the EM parameters and their temporal fluctuations, the properties of existing interface boundaries and those of the target and its state of motion. While virtually all wave statistics are relevant under appropriate circumstances, our attention here is focused upon the temporal spectrum of the target return*, a quantity of direct interest for design and operational considerations of radar systems with doppler capabilities. The spectral modifications of a narrow band signal are traceable to two first order factors: (a) the time dependence characterizing the fluctuating medium (the vegetation slab in our application) and (b) the motion of the target through a randomly inhomogeneous field, with the latter likely to dominate.

The objective of this analytical work is to relate the desired spectral information to the presumably known statistical measures which characterize the random slab (specifically its space-time correlation function or the corresponding Fourier transform). Several avenues are open to us within the framework of the previously introduced continuum model[(1)]. Of these, the simplest alternative, that involving a single random scatter event of either the incident or the return wave, is to be pursued. The included effects are depicted symbolically in Fig. 1, in which refractive and ground effects are shown and are indeed accounted for in the analytical sequence that follows. The methodology resembles that presented in Ref. (1). Conclusions and

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*The analogous problem concerning the spectrum and other statistical properties associated with the clutter return have been investigated in some detail. The results and conclusions have been reported in Ref. 1.

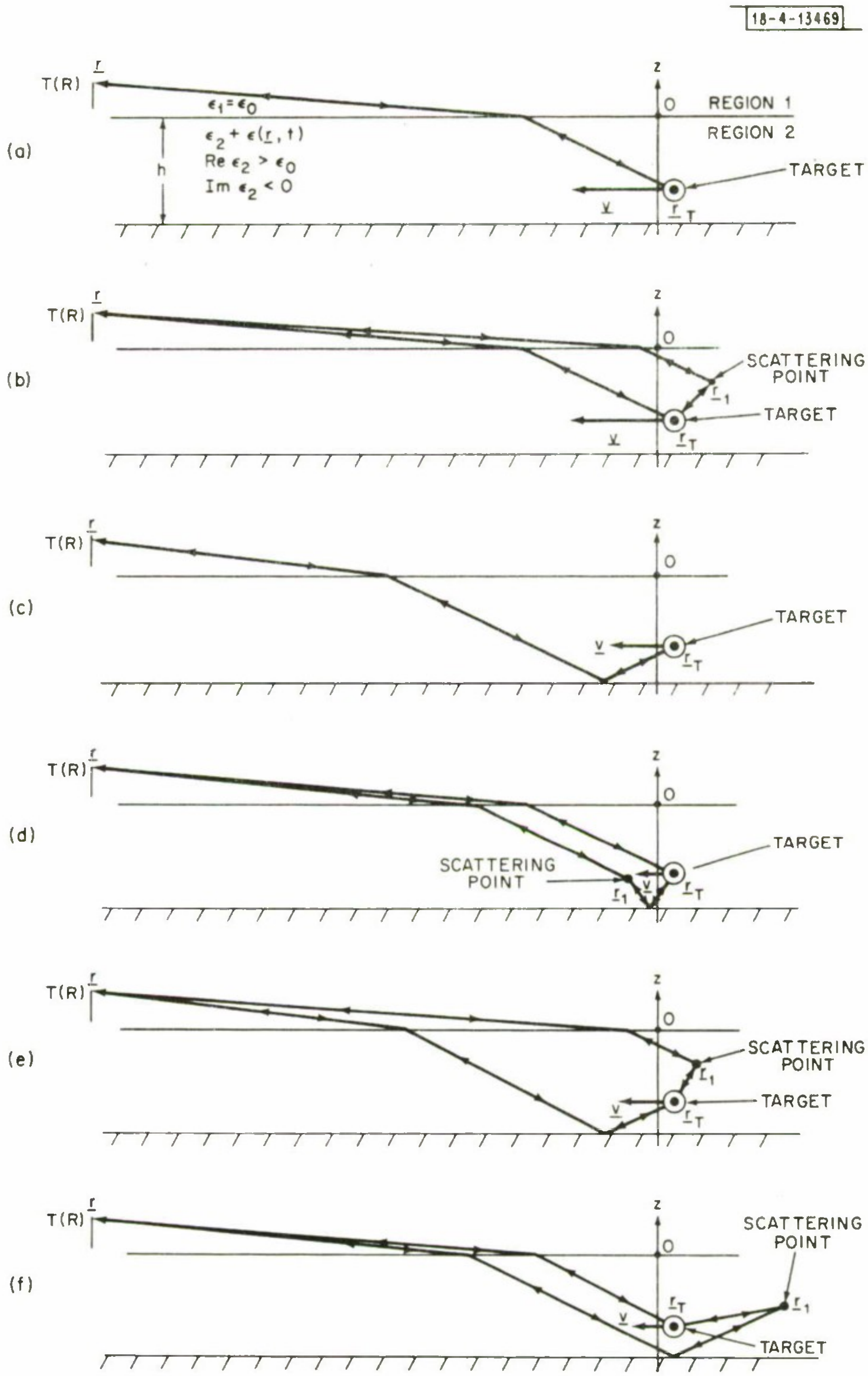

Fig. 1. Ray configurations.

results presented in it are not repeated*.

The conceptual simplicity inherent in the (distorted wave) single scatter theory to be pursued does not fully propagate through the analytical sequence which is rather laborious. However, through the imposition of various simplfying assumptions, most of which not too objectionable, the final result (Eqs. (68 and 76)) is analytically simple, numerically useful and its physical content readily interpretable. In fact, a crude but direct and simple heuristic construction of the final result is presented in Sec. F. It should be mentioned that the presented analysis retains its validity and applicability beyond the physical context of immediate interest.

The problem at hand is described and rigorously formulated in Sec. B where a formal expression (14) for the target generated electric field ($\underline{E}_T$) at the receiving antenna is derived. An explicit expression for the stochastic target return is obtained in Sec. C consistently with the "distorted wave" Born approximation in which background refraction, ground effects and single incoherent scattering are properly accounted for. The temporal correlation coefficient ($C_E(\tau)$) and the associated spectral density are calculated in Sec. D. The main result (68) is obtained subject to the omission of the motion associated with the randomly distributed scattering centers. Sec. D terminates with several interpreting notes. The spectral perturbation associated with the random motion of the scattering centers is presented in Sec. E. The main result is contained in Eq. (76). The content of Sec. F has been mentioned

*Reference to equations in Ref. (1) are made by adding I ( ) in front of the equation's number.

above. The mean power return is calculated in Sec. G and the non-trivial question regarding the definition of the effective scattering volume is the subject matter of Appendix A.

B. FORMULATION AND DESCRIPTION OF THE PROBLEM

The physical configuration comprises a "point" target moving at a constant velocity ($\underline{v}$) through a randomly fluctuating slab region depicted in Fig. 1. The slab, modeling the vegetation, is parameterized electromagnetically by random permittivity fluctuations ($\epsilon(\underline{r}, t)$) superimposed on a lossy background represented by the complex permittivity ϵ_2, consistently with the "continuum" model presented in Ref. (1). A heuristic sequence arguing the validity of the continuum model is also given in Ref. (1) and need not be repeated. The target is characterized by an effective point current (dipole) element

$$\underline{J}_T(\underline{r}) = j\omega_o \underline{P}_T = \beta \underline{E}_2(\underline{r}) \delta(\underline{r} - \underline{r}_T(t)) \tag{1}$$

where $\underline{r}_T(t)$ denotes the target location, and $\beta/j\omega_o$ is the target polarizability (e.g. $\beta = -\frac{4\pi}{\mu_o k_2}(k_2 a)^3 \frac{\epsilon_r - 1}{\epsilon_r + 2}$ for a small dielectric sphere of radius $a << \lambda$ and relative permittivity ϵ_r; k_2 the wave number in region 2). The electric fields $\underline{E}_1(\underline{r}, t)$ and $\underline{E}_2(\underline{r}, t)$ in regions 1 and 2 respectively are solutions of the vector wave equations.

$$\nabla \times \nabla \times \underline{E}_1 - k_1^2 \underline{E}_1 = j\omega_o \mu_o \underline{J}_1(\underline{r}), \quad z > 0 \tag{2}$$

$$\nabla \times \nabla \times \underline{E}_2 - k_2^2 \underline{E}_2 - \omega_o^2 \mu_o \epsilon(\underline{r}, t) \underline{E}_2 = -j\omega_o \mu_o \underline{J}_T, \quad z < 0 \tag{3}$$

together with the continuity of the transverse electric and magnetic fields at $z = 0$, the vanishing of the transverse electric field at $z = -h$ (h denoting the

effective slab thickness) and the radiation condition at $z \to \infty$. $\underline{J}_1$ in (2) denotes the exciting current density situated in region 1. With the Green's functions ($\underset{\sim}{G}_{1,2}(\underline{r}, \underline{r}')$) defined by

$$\nabla \times \nabla \times \hat{\underset{\sim}{G}}_1 - k_1^2 \hat{\underset{\sim}{G}}_1 = 0, \quad z > 0 \tag{4}$$

$$\nabla \times \nabla \times \hat{\underset{\sim}{G}}_2 - k_2^2 \hat{\underset{\sim}{G}}_2 - \omega_o^2 \mu_o \epsilon(\underline{r}, t) \hat{\underset{\sim}{G}}_2 = \underset{\sim}{I}\, \delta(\underline{r} - \underline{r}'), \; 0 < z < h \tag{5}$$

Eqs. (2, 3) may be formally inverted. One obtains,

$$\underline{E}_{\substack{1\\2}\,T}(\underline{r}\,|\,t) = -j\omega_o \mu_o \int d^3 r_1\, \hat{\underset{\sim}{G}}_{\substack{1\\2}}(\underline{r}, \underline{r}_1) \cdot \underline{J}_T(\underline{r}_1)$$

$$= -j\omega_o \mu_o \beta\, \hat{\underset{\sim}{G}}_{\substack{1\\2}}(\underline{r}, \underline{r}_T(t)\,|\,t) \cdot \underline{E}_2(\underline{r}_T(t)\,|\,t) \tag{6}$$

where $\underline{E}_T$ is the target generated field defined by

$$\underline{E}_{1,2T} = \underline{E}_{1,2} - \underline{E}_{1,2}(\underline{J}_T = 0) \tag{7}$$

The difficulty in (6) stems from the fact that neither $\hat{\underset{\sim}{G}}_{1,2}$ nor $\underline{E}_2$ are known. The ultimate validity of the theory depends on the accuracy with which these quantities can be estimated.

We now focus attention on $\hat{\underset{\sim}{G}}_{1,2}$. Let,

$$\hat{\underset{\sim}{G}}_{\substack{1\\2}}(\underline{r}, \underline{r}_1\,|\,t) = \underset{\sim}{G}_{\substack{1\\2}}(\underline{r}, \underline{r}_1) + \underset{\sim}{g}_{\substack{1\\2}}(\underline{r}, \underline{r}_1\,|\,t) \tag{8}$$

where $\underset{\sim}{G}_{\substack{1\\2}} = \hat{\underset{\sim}{G}}_{\substack{1\\2}}(\epsilon = 0)$ and $\underset{\sim}{g}_{\substack{1\\2}}$ is calculated via the "distorted wave" Born approximation. We have (exactly)

$$\nabla \times \nabla \times \underset{\sim}{g}_1 - k_1^2 \underset{\sim}{g}_1 = 0 \tag{9}$$

$$\nabla \times \nabla \times \underset{\sim}{g}_2 - k_2^2 \underset{\sim}{g}_2 = \omega_o^2 \mu_o \epsilon \hat{\underset{\sim}{G}}_2 \tag{10}$$

which, upon inversion, results in the integral equation

$$\underset{\sim}{g}_{\substack{1\\2}}(\underline{r}, \underline{r}' \mid t) = \omega_o^2 \mu_o \int_V d^3 r_1 \, \epsilon(\underline{r}_1, t) \, \underset{\sim}{G}_{\substack{1\\2}}(\underline{r}, \underline{r}_1) \cdot \hat{\underset{\sim}{G}}_2(\underline{r}_1, \underline{r}' \mid t) \tag{11}$$

where a detailed definition of the effective scattering volume (V) is presented in Appendix A. Consistently, with the distorted wave Born approximation, the unknown $\hat{\underset{\sim}{G}}_2$ in (11) is replaced by $\underset{\sim}{G}$ which is, in turn, determined from the deterministic background problem[1]. The field $\underline{E}_2$ appearing in (6) may be written as

$$\underline{E}_2(\underline{r} \mid t) = \underline{E}_2^{(o)}(\underline{r}) + \underline{E}_{2s}(\underline{r} \mid t) \tag{12}$$

where $\underline{E}_2^{(o)} = \underline{E}_2(\epsilon = 0)$ and

$$\underline{E}_{2s}(\underline{r} \mid t) = \omega_o^2 \mu_o \int_V d^3 r_1 \, \epsilon(\underline{r}_1, t) \, \underset{\sim}{G}(\underline{r}, \underline{r}_1) \cdot \underline{E}_2(\underline{r}_1 \mid t) \tag{13}$$

Once again, consistently with the distorted wave Born approximation, $\underline{E}_2$ in (13) is to be replaced by $\underline{E}_2^{(o)}$. The substitution of Eqs. (8, 12) into (6) results in

$$\underline{E}_{\substack{1\\2}T}(\underline{r} \mid t) = -j\omega_o \mu_o \beta \Big[\underset{\sim}{G}_{\substack{1\\2}}(\underline{r}, \underline{r}_T) \cdot \underline{E}_2^{(o)}(\underline{r}_T) + \underset{\sim}{g}_{\substack{1\\2}}(\underline{r}, \underline{r}_T \mid t) \cdot \underline{E}_2^{(o)}(\underline{r}_T)$$

$$+ \underset{\sim}{G}_{\substack{1\\2}}(\underline{r}, \underline{r}_T) \cdot \underline{E}_{2s}(\underline{r}_T \mid t) \Big] \tag{14}$$

where each of the three terms is readily identified with the distinct contributions depicted in Figs. 1a and 1b, and the double scatter term has been omitted.

C. THE TARGET RETURN

Eq. (14) may be cast into the form

$$\underline{E}_{\substack{1\\2}T}(\underline{r} \mid t) = \underline{E}_{\substack{1\\2}T}^{(o)}(\underline{r} \mid t) + \Delta \underline{E}_{\substack{1\\2}T}(\underline{r} \mid t) \tag{15}$$

where

$$\underset{2}{\underline{E}_{1T}^{(o)}}(\underline{r}\,|\,t) = -j\omega_o \mu_o \beta \underset{2}{\underset{\sim}{G}_1}(\underline{r}, \underline{r}_T) \cdot \underline{E}_2^{(o)}(\underline{r}_T) \tag{16}$$

and

$$\Delta \underset{2}{\underline{E}_{1T}}(\underline{r}\,|t) = -j\omega_o^3 \mu_o^2 \beta \int_V d^3 r_1 \, \epsilon(\underline{r}_1, t) \, [\underset{2}{\underset{\sim}{G}_1}(\underline{r}, \underline{r}_1) \cdot \underset{\sim}{G}_2(\underline{r}_1, \underline{r}_T) \cdot \underline{E}_2^{(o)}(\underline{r}_T)$$

$$+ \underset{2}{\underset{\sim}{G}_1}(\underline{r}, \underline{r}_T) \cdot \underset{\sim}{G}_2(\underline{r}_T, \underline{r}_1) \cdot \underline{E}_2^{(o)}(\underline{r}_1)\,] \tag{17}$$

It is observed that the term $\underset{2}{\underline{E}_{1T}^{(o)}}$ corresponds to a return from a point target moving at a constant velocity through a uniformly illuminated region*. It, therefore, represents a spectrally unperturbed (with the exception of a simple shift) return. $\underline{E}_2^{(o)}$ and $\underset{\sim}{G}_1$ are taken to be these derived in Ref. (1) (Eqs. I(98) and I(101), respectively)

$$\underset{\sim}{G}_1(\underline{r}, \underline{r}_1) \sim \frac{z\,[\underset{\sim}{1} - \underline{\rho}_o \, \rho_o]}{2\pi \sqrt{\eta^2 - 1}\, L_1^2} \exp\left\{-jk_1 \left[|\underline{\rho} - \underline{\rho}_1| + \frac{1}{2}\frac{z^2}{|\underline{\rho} - \underline{\rho}_1|} - \sqrt{\eta^2 - 1}\, z_1\right] + \alpha \frac{z_1}{\sqrt{1 - \eta^{-2}}}\right\} \tag{18}$$

$$\underline{E}_2^{(o)}(\underline{r}_1) \sim \underline{\varphi}_o \frac{2z\, F(\varphi)}{\sqrt{\eta^2 - 1}\, L_1^2} \exp\left\{-jk_1 \left[|\underline{\rho} - \underline{\rho}_1| + \frac{1}{2}\frac{z^2}{|\underline{\rho} - \underline{\rho}_1|} - \sqrt{\eta^2 - 1}\, z_1\right] + \alpha \frac{z_1}{\sqrt{1 - \eta^{-2}}}\right\} \tag{19}$$

The dyadic Green's function $\underset{\sim}{G}_2$ contains direct ray contributions as well as contributions associated with interface (ground vegetation and air vegetation) reflections. For $\underset{\sim}{G}_2$ air vegetation interface reflections may be safely ignored

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* The transverse field inhomogeneity associated with the antenna beam may be disregarded.

for two reasons. Firstly, the interface discontinuity is small and with the exception of critical incidence so is the associated reflection coefficient. Secondly, since the medium is lossy and since the anticipated targets are situated near ground, the reflected contribution is further attenuated due to the absorption and scattering losses and become insignificant even for larger interface discontinuity. With ground effects included, one has

$$\underset{\sim}{G}_2(\underline{r}, \underline{r}') \sim \frac{\exp\left[-jk_2 |\underline{r} - \underline{r}'| - \alpha |\underline{r} - \underline{r}'|\right]}{4\pi |\underline{r} - \underline{r}'|} (\underset{\sim}{1} - \underline{r}_o \underline{r}_o), \quad \underline{r}_o = \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|} \tag{20}$$

and

$$\underset{\sim}{G}_{2R}(\underline{r}, \underline{r}') \sim - \frac{\exp\{(-jk_2 - \alpha)\left[|\underline{\rho} - \underline{\rho}'|^2 + (z + z' + 2h)^2\right]^{1/2}\}}{4\pi\left[|\underline{\rho} - \underline{\rho}'|^2 + (z + z' + 2h)^2\right]^{1/2}} (\underset{\sim}{1} - \underline{r}_{o1} \underline{r}_{o1}),$$

$$\underline{r}_{o1} = \frac{\underline{r} - \underline{r}''}{|\underline{r} - \underline{r}''|} \tag{21}$$

where $\underline{r}''$ denotes the image-point location (Fig. 2). Ground effects are accounted for by replacing $\underset{\sim}{G}_1$ and $\underline{E}_2^{(o)}$ in Eq. (17) by $\underset{\sim}{G}_{\substack{1\\2}} + \underset{\sim}{G}_{\substack{1R\\2}}$ and $\underline{E}_2^{(o)} + \underline{E}_{2R}^{(o)}$, respectively. The subscript R denotes ground reflections which for the horizontally polarized constituents are obtained by a phase reversal and the replacement of $-z_1$ and $-z_T$ by $2h + z_1$ and $2h + z_T$, respectively. It has been shown in Ref. (1) that single ground reflections are properly accounted for by a multiplicative correction factor

$$\gamma(z_1) = 1 - \exp\left[\left(-jk_1 \sqrt{\eta^2 - 1} - \frac{2\alpha}{\sqrt{1 - \eta^{-2}}}\right)(h + z_1)\right] \tag{22}$$

i. e.,

$$\underline{E}_2^{(o)}(\underline{r}_1) + \underline{E}_{2R}^{(o)}(\underline{r}_1) \approx \gamma(z_1)\, \underline{E}_2^{(o)}(\underline{r}_1) \tag{23}$$

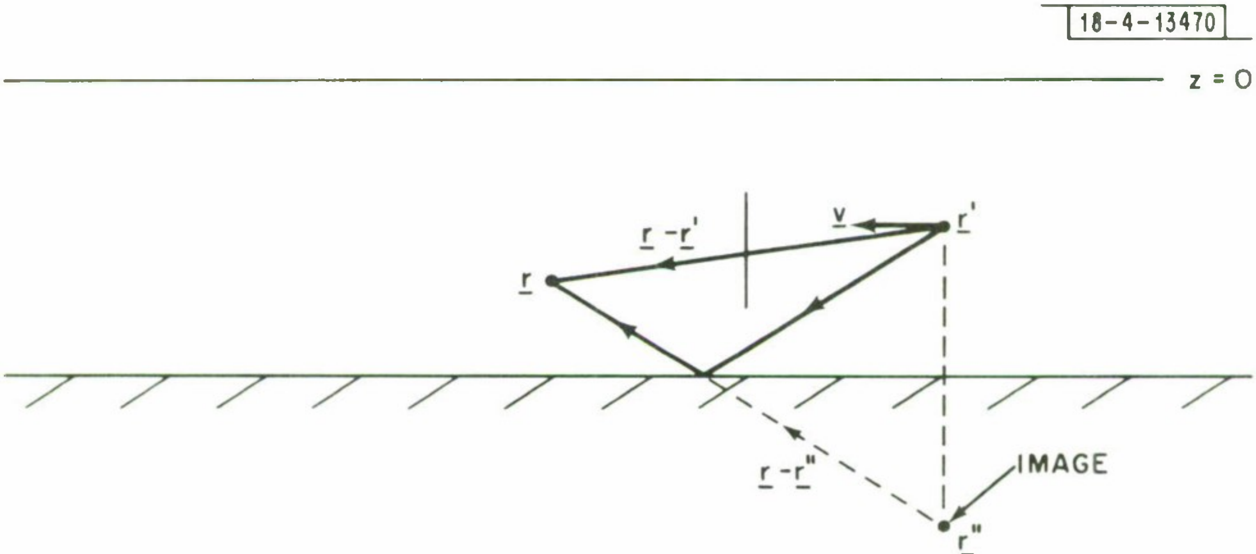

Fig. 2. Ray configuration relevant to the calculation of G_2.

and

$$\underset{\sim}{G}_1 (\underline{r}, \underline{r}_1) + \underset{\sim}{G}_{1R} (\underline{r}, \underline{r}_1) \approx \gamma (z_1) \underset{\sim}{G}_1 (\underline{r}, \underline{r}_1) \tag{24}$$

where $\underset{\sim}{G}_1$ and $\underline{E}_2^{(o)}$ are given by Eqs. (18, 19) respectively. Strictly speaking, neither Eq. (21) nor Eq. (24) are correct since vertically polarized field constituents do not experience a phase inversion upon reflection. These results, however, are applicable in our case since both transmission and reception are horizontally polarized and depolarization due to either coherent or incoherent scattering is expected to be small[1].

In the analytical sequence which follows, ground contributions associated with $\underset{\sim}{G}_{2R}$ (Figure 1d) are ignored for the sake of convenience. These contributions may be readily accounted for by a procedure identical to that presented below. However, since ground effects are of secondary importance to the determination of the spectrum, as is demonstrated below, we do not return to investigate this contribution. From Eqs. (18, 19, 20, 23 and 24) one has

$$[\underset{\sim}{G}_1 (\underline{r}, \underline{r}_1) + \underset{\sim}{G}_{1R}(\underline{r}, \underline{r}_1)] \cdot \underset{\sim}{G}_2 (\underline{r}_1, \underline{r}_T) \cdot [\underline{E}_2^{(o)} (\underline{r}_T) + \underline{E}_2^{(o)} (\underline{r}_T)] \sim$$

$$\underline{p} \frac{2 z^2 \gamma (z_1) \gamma (z_T) F(\varphi)}{8\pi^2 (\eta^2 - 1) \rho_o^4 |\underline{r}_1 - r_T|} \exp \Big\{ -jk_1 [|\underline{\rho} - \underline{\rho}_1| + |\underline{\rho} - \underline{\rho}_T| + \frac{z^2}{\rho_o} - \sqrt{\eta^2 - 1}(z_1 + z_T)]$$

$$-jk_2 |\underline{r}_1 - \underline{r}_T| + \alpha \frac{z_1 + z_T}{\sqrt{1 - \eta^{-2}}} - \alpha |\underline{r}_1 - \underline{r}_T| \Big\} \tag{25}$$

where,

$$\underline{p} = [\underset{\sim}{1} - \frac{(\underline{r} - \underline{r}_1)(\underline{r} - \underline{r}_1)}{|\underline{r} - \underline{r}_1|^2}] \cdot [\underset{\sim}{1} - \frac{(\underline{r}_1 - \underline{r}_T)(\underline{r}_1 - \underline{r}_T)}{|\underline{r}_1 - \underline{r}_T|^2}] \cdot \underline{\varphi}_o \tag{26}$$

If (a) $|\underline{r}| >> |\underline{r}_1 - \underline{r}_T|$ and (b) ignoring vertical (crossed coupled) components not sensed by the receiver one obtains (see Appendix B and Fig. 3)

$$\underline{p} \approx \underline{\varphi}_o \sin^2 \chi \qquad (\cos \chi = \underline{\varphi}_o \cdot \frac{\underline{r}_1 - \underline{r}_T}{|\underline{r}_1 - \underline{r}_T|}) \tag{27}$$

Furthermore, in the transition from Eqs. (18, 19) to Eq. (25) the geometric-optical path (L_1) has been replaced by the range (ρ_o) in the amplitude factor (see Fig. 3). Similarly, corresponding to the second term in the integrand of Eq. (17),

$$[\underset{\sim}{G}_1 (\underline{r}, \underline{r}_T) + \underset{\sim}{G}_{1R} (\underline{r}, \underline{r}_T)] \cdot \underset{\approx}{G}_2 (\underline{r}_T, \underline{r}_1) \cdot [\underline{E}_2^{(o)} (\underline{r}_1) + \underline{E}_{2R}^{(o)} (\underline{r}_1)] \sim$$

$$\hat{\underline{p}} \frac{2z^2 \gamma(z_1) \gamma(z_T) F(\varphi)}{8\pi^2 (\eta^2 - 1) \rho_o^4 |\underline{r}_1 - \underline{r}_T|} \exp\left\{-jk_1 [|\underline{\rho} - \underline{\rho}_1| + |\underline{\rho} - \underline{\rho}_T| + \frac{z^2}{\rho_o} - \sqrt{\eta^2 - 1} (z_1 + z_T)]\right.$$

$$\left. -jk_2 |\underline{r}_1 - \underline{r}_T| + \alpha \frac{z_1 + z_T}{\sqrt{1 - \eta^{-2}}} - \alpha |\underline{r}_1 - \underline{r}_T| \right\} \tag{28}$$

Under conditions (a) and (b) above, $\underline{p} \approx \hat{\underline{p}}$ and expressions (25) and (28) are identical. The substitution of Eqs. (25, 28) into Eq. (17) together with the approximate forms

$$|\underline{\rho} - \underline{\rho}_1| \approx \rho_o - \underline{\rho}_o \cdot \underline{\rho}_1, \quad |\underline{\rho} - \underline{\rho}_T| \approx \rho_o - \underline{\rho}_o \cdot \underline{\rho}_T \tag{29}$$

results in

$$\Delta \underline{E}_T (\underline{r} | t) \sim -j\omega_o^3 \mu_o^2 \beta \underline{\varphi}_o \frac{z^2 \gamma(z_T) F(\varphi)}{2\pi^2 (\eta^2 - 1) \rho_o^4} \exp\left\{-2jk_1 \rho_o - jk_1 \frac{z^2}{\rho_o} + jk_1 \sqrt{\eta^2 - 1}\, z_T + \right.$$

$$\left. \alpha \frac{z_T}{\sqrt{1 - \eta^{-2}}} + jk_1 \underline{\rho}_o \cdot \underline{\rho}_T \right\} I (\underline{r}, t) \tag{30}$$

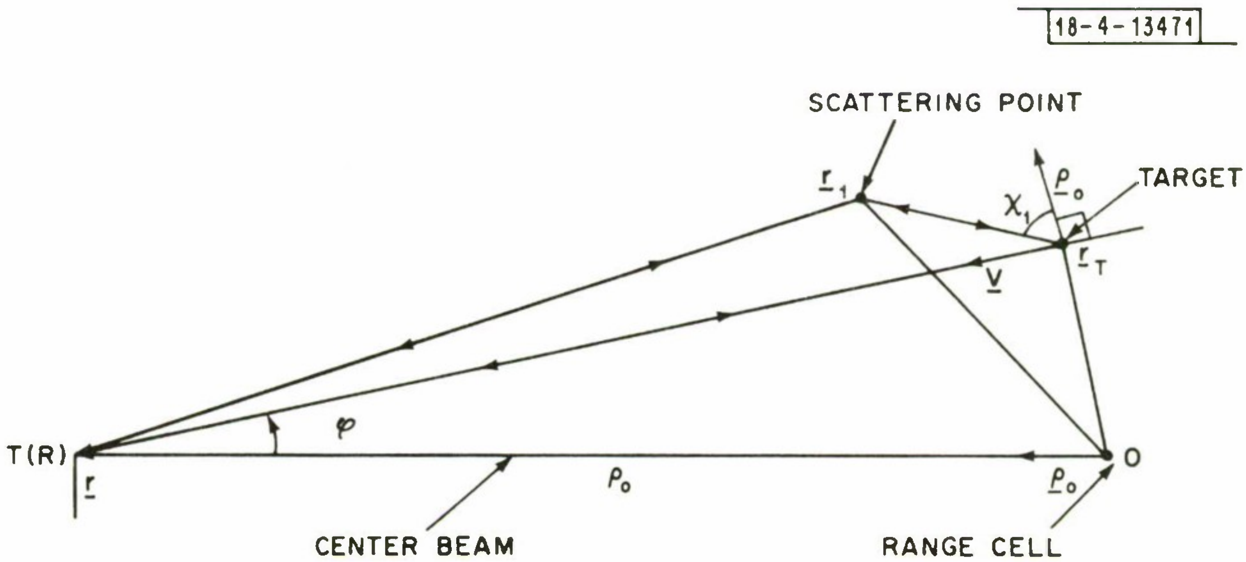

Fig. 3. Top view.

where

$$I(\underline{r}, t) = \int_V d^3 r_1 \; \epsilon(\underline{r}_1, t) \sin^2 \chi \gamma(z_1) \frac{1}{|\underline{r}_1 - \underline{r}_T|} \exp\left\{ jk_1 \underline{\rho}_o \cdot \underline{\rho}_1 + \alpha \frac{z_1}{\sqrt{1 - \eta^{-2}}} \right.$$

$$\left. + jk_1 \sqrt{\eta^2 - 1} \; z_1 - (jk_2 + \alpha) |\underline{r}_1 - \underline{r}_T| \right\} \tag{31}$$

The plane wave expansion

$$|\underline{r}_1 - \underline{r}_T|^{-1} \exp[-j\hat{k}_2 |\underline{r}_1 - \underline{r}_T|] = \frac{1}{2\pi^2} \int d^3\kappa \frac{\exp[-j\underline{\kappa} \cdot (\underline{r}_1 - \underline{r}_T)]}{\kappa^2 - \hat{k}_2^2}, \quad \hat{k}_2 = k_2 - j\alpha \tag{32}$$

will be found convenient in the forthcoming analytical sequence. For any plane wave constituent (characterized by the wave number $\underline{\kappa}$) the angle χ is constant depending exclusively on $\underline{\kappa}$. The substitution of Eq. (32) into (31) results in

$$I = \frac{1}{2\pi^2} \int d^3\kappa \frac{\exp[j\underline{\kappa} \cdot \underline{r}_T]}{\kappa^2 - \hat{k}_2^2} \sin^2 \chi(\kappa) \int d^3 r_1 \; \epsilon(\underline{r}_1, t) \gamma(z_1)$$

$$\exp\left\{ -j[\underline{\kappa} - k_1 \underline{\rho}_o - k_1 \sqrt{\eta^2 - 1} \; \underline{z}_o] \cdot \underline{r}_1 \right\} \tag{33}$$

Eq. (30), whenever applicable, constitutes a convenient starting point for a statistical analysis of the backscattered wave. Of primary interest are the correlation properties of the return signal (both spatial and temporal), the corresponding spectra and the mean return power. The next section is dedicated to the study of the temporal correlation and the associated spectrum. A simple and readily interpretable result is derived.

D. THE TEMPORAL CORRELATION COEFFICIENT: $C_E(\tau)$

The field temporal correlation coefficient is defined by

$$<|\underline{E}_{1T}(\underline{r},t)|^2> C_E(\tau) = <\underline{E}_{1T}(\underline{r},t)\cdot \underline{E}^*_{1T}(\underline{r},t-\tau)> = \underline{E}^{(o)}_{1T}(\underline{r},t)\cdot \underline{E}^{(o)*}_{1T}(\underline{r},t-\tau)$$

$$+ <\Delta \underline{E}_{1T}(\underline{r},t)\cdot \Delta \underline{E}^*_{1T}(\underline{r},t-\tau)> \tag{34}$$

Explicitly,

$$\underline{E}^{(o)}_{1T}(t)\cdot \underline{E}^{(o)*}_{1T}(t-\tau) = \omega_o^2\,\mu_o^2\,|\beta|^2\,\frac{z^4|\gamma(z_T)|^4|F(\varphi)|^2}{\pi^2(\eta^2-1)^2\rho_o^8}\exp[j2k_1\underline{\rho}_o\cdot\underline{v}\tau + \frac{4\alpha z_T}{\sqrt{1-\eta^{-2}}}] \tag{35}$$

and

$$<\Delta\underline{E}_{1T}(t)\cdot\Delta\underline{E}^*_{1T}(t-\tau)> = \omega_o^6\,\mu_o^4\,|\beta|^2\,\frac{z^4|\gamma(z_T)|^2F(\varphi)|^2}{4\pi^4(\eta^2-1)^2\rho_o^8}<I(t)\,I^*(t-\tau)>$$

$$\exp[jk_1\,\underline{\rho}_o\cdot\underline{v}\tau + 2\alpha\frac{z_T}{\sqrt{1-\eta^{-2}}}] \tag{36}$$

where the relation

$$\underline{r}_T(t) - \underline{r}_T(t-\tau) = \underline{v}\,\tau \tag{37}$$

has been utilized. Consequently, one has

$$<|\underline{E}_{1T}|^2> C_E(\tau) = \omega_o^2\,\mu_o^2\,|\beta|^2\,\frac{z^4|\gamma(z_T)|^4|F(\varphi)|^2}{\pi^2(\eta^2-1)^2\rho_o^8}\left\{1+\frac{\omega_o^4\mu_o^2}{4\pi^2|\gamma(z_T)|^2}\right.$$

$$\left.<I(t)\,I^*(t-\tau)>\exp[-jk_1\underline{\rho}_o\cdot\underline{v}\tau - 2\alpha\frac{z_T}{\sqrt{1-\eta^{-2}}}]\exp[j2k_1\underline{\rho}_o\cdot\underline{v}\tau + \frac{2\alpha z_T}{\sqrt{1-\eta^{-2}}}]\right\} \tag{38}$$

with

$$<I(t)\,I^*(t-\tau)> = \frac{1}{4\pi^4}\int d^3\kappa_1\int d^3\kappa_2\,\frac{\sin^2\chi(\underline{\kappa}_1)\,\sin^2\chi(\underline{\kappa}_2)}{[\kappa_1^2-\hat{k}_2^2][\kappa_2^2-\hat{k}_2^{*2}]}$$

$$\exp j[\underline{\kappa}_1\cdot\underline{r}_T(t) - \underline{\kappa}_2\cdot\underline{r}_T(t-\tau)]\int_V d^3r_1\int_v d^3r_2\,C_\epsilon(\underline{r}_1-\underline{r}_2|\tau)\gamma(z_1)\gamma^*(z_2)$$

$$\exp\left\{-j\underline{\kappa}_1\cdot\underline{r}_1 + j\underline{\kappa}_2\cdot\underline{r}_2 + jk_1\underline{\rho}_o\cdot(\underline{r}_1-\underline{r}_2) + jk_1\sqrt{\eta^2-1}\,(z_1-z_2) + \frac{\alpha(z_1+z_2)}{\sqrt{1-\eta^{-2}}}\right\} \tag{39}$$

and

$$C_\epsilon(\underline{r}_1 - \underline{r}_2 | \tau) = \langle \epsilon(\underline{r}_1, t)\, \epsilon^*(\underline{r}_2, t - \tau) \rangle$$

$$= \int \phi_\epsilon(\underline{\varkappa} | \tau)\, e^{-j\underline{\varkappa} \cdot (\underline{r}_1 - \underline{r}_2)}\, d^3\varkappa \tag{40}$$

Eq. (40) is based on the assumption of statistical homogeniety and stationarity of the process $\epsilon(\underline{r}, t)$. The following change of variables is introduced for convenience:

$$\hat{\underline{r}} = \underline{r}_1 - \underline{r}_2,\ \underline{R} = \frac{1}{2}(\underline{r}_1 + \underline{r}_2) \text{ or } \underline{r}_{\substack{1\\2}} = \underline{R} \pm \frac{1}{2}\hat{\underline{r}}$$

$$\hat{\underline{\varkappa}} = \underline{\varkappa}_1 - \underline{\varkappa}_2,\ \underline{\varkappa} = \frac{1}{2}(\underline{\varkappa}_1 + \underline{\varkappa}_2) \text{ or } \underline{\varkappa}_{\substack{1\\2}} = \bar{\underline{\varkappa}} \pm \frac{1}{2}\hat{\underline{\varkappa}}$$

$$\hat{\underline{r}}_T = \underline{r}_T(t) - \underline{r}_T(t-\tau) = \underline{v}\tau,\ \bar{\underline{r}}_T = \frac{1}{2}[\underline{r}_T(t) + \underline{r}_T(t-\tau)]$$

or

$$\underline{r}_T(t-\tau) = \bar{\underline{r}}_T - \frac{\underline{v}\tau}{2},\ \underline{r}_T(t) = \bar{\underline{r}}_T + \frac{\underline{v}\tau}{2} \tag{41}$$

$$\hat{z} = z_1 - z_2,\ Z = \frac{1}{2}(z_1 + z_2) \text{ or } z_{\substack{1\\2}} = Z \pm \frac{1}{2}\hat{z}$$

and Eq. (39) becomes

$$\langle I(t)\, I^*(t-\tau) \rangle = \frac{1}{4\pi^4} \int d^3\bar{\underline{\varkappa}} \int d^3\hat{\underline{\varkappa}}\ \frac{\sin^2\chi_1 \sin^2\chi_2}{\left[\left|\bar{\underline{\varkappa}} + \frac{1}{2}\underline{\varkappa}\right|^2 - k_2^2\right]\left[\left|\bar{\underline{\varkappa}} - \frac{1}{2}\underline{\varkappa}\right|^2 - k_2^{*2}\right]}$$

$$\exp\left[j\bar{\underline{\varkappa}} \cdot \underline{v}\tau + j\hat{\underline{\varkappa}} \cdot \bar{\underline{r}}_T\right] \int_V d^3R \int_V d^3\hat{r}\, C_\epsilon(\hat{\underline{r}} | \tau)\, \gamma\left(Z + \frac{1}{2}\hat{z}\right) \gamma^*\left(Z - \frac{1}{2}\hat{z}\right)$$

$$\exp\left\{j\left[-\bar{\underline{\varkappa}} \cdot \hat{\underline{r}} - \hat{\underline{\varkappa}} \cdot \underline{R} + k_1\, \underline{\rho}_o \cdot \hat{\underline{r}} + k_1 \sqrt{\eta^2 - 1}\, \hat{z}\right] + \frac{2\alpha Z}{\sqrt{1 - \eta^{-2}}}\right\} \tag{42}$$

here,

$$\chi_{\substack{1\\2}} = \chi\left(\bar{\underline{\varkappa}} \pm \frac{1}{2}\hat{\underline{\varkappa}}\right) \tag{43}$$

For simplicity, we further assume that γ does not vary substantially over the correlation length (a), i. e.

$$k_1 \alpha \sqrt{\eta^2 - 1} << 1, \qquad \frac{\alpha}{\sqrt{1 - \eta^{-2}}} a << 1 \tag{44}$$

so that the approximate relationship

$$\gamma (Z + \frac{1}{2} z) \gamma^* (Z - \frac{1}{2} z) \approx | \gamma (Z) |^2 \tag{45}$$

may be utilized in conjunction with (42). To the same order we may also ignore the refractive phase term $k_1 \sqrt{\eta^2 - 1}\, z$ in Eq. (42). It is noted, however, that this approximation is unessential and may be relaxed if necessary. With (45) in mind and for an effective scattering volume characterized by a linear dimension large compared with the correlation length, Eq. (42) yields

$$< I(t)\, I^*(t - \tau) > = \frac{(2\pi)^3}{4\pi^4} \int d^3\bar{\varkappa} \int d^3 \hat{\varkappa} \frac{\sin^2 \chi_1 \sin^2 \chi_2}{[\, | \underline{\bar{\varkappa}} + \frac{1}{2} \underline{\hat{\varkappa}} |^2 - \hat{k}_2^2][\, | \underline{\bar{\varkappa}} - \frac{1}{2} \underline{\hat{\varkappa}} |^2 - \hat{k}_2^{*2}]}$$

$$F(\underline{\hat{\varkappa}})\, \phi_\epsilon (k_1 \underline{\rho}_o + k_1 \sqrt{\eta^2 - 1}\, \underline{\rho}_o - \underline{\bar{\varkappa}} \,|\, \tau) \exp [j \underline{\bar{\varkappa}} \cdot \underline{v} \tau + j \underline{\hat{\varkappa}} \cdot \underline{\bar{r}}_T] \tag{46}$$

where,

$$F(\underline{\hat{\varkappa}}) = \int d^3 R \, | \gamma (Z) |^2 e^{j \underline{\hat{\varkappa}} \cdot \underline{R} + \frac{2 \alpha Z}{\sqrt{1 - \eta^{-2}}}} \tag{47}$$

and ϕ_ϵ is the spatial spectral density defined in Eq. (40). Eq. (46) may be cast into the form

$$< I(t)\, I^*(t - \tau) > = \frac{2}{\pi} \exp [j(k_1 \underline{\rho}_o + k_1 \sqrt{\eta^2 - 1}\, \underline{z}_o) \cdot \underline{v} \tau] \int d^3 \hat{\varkappa}\, F(\underline{\hat{\varkappa}}) M(\underline{\hat{\varkappa}} \,|\, \tau)$$

$$\exp [j \underline{\hat{\varkappa}} \cdot \underline{\bar{r}}_T] \tag{48}$$

where

$$M(\underline{\hat{\varkappa}} \mid \tau) = \int d^3 \varkappa \frac{\phi_\epsilon(\underline{\varkappa} \mid \tau) \sin^2 \chi_1 \sin^2 \chi_2 \exp[-j\underline{\varkappa} \cdot \underline{v}\tau]}{[\,|k_2 \rho_o - \underline{\varkappa} + \frac{1}{2}\underline{\hat{\varkappa}}|^2 - \hat{k}_2^{\,2}][\,|k_2 \rho_o - \underline{\varkappa} - \frac{1}{2}\underline{\hat{\varkappa}}|^2 - \hat{k}_2^{\,*2}]}, \tag{49}$$

$$\chi_{\frac{1}{2}} = \chi(k_2 \rho_o - \underline{\varkappa} \pm \frac{1}{2}\underline{\hat{\varkappa}}) \tag{50}$$

and a new variable

$$\underline{\varkappa} = k_1 \rho_o + k_1 \sqrt{\eta^2 - 1}\ \underline{z}_o - \underline{\bar{\varkappa}} \approx k_2 \rho_o - \underline{\bar{\varkappa}} \tag{51}$$

has been introduced. The parameter $k_1 \rho_o + k_1 \sqrt{\eta^2 - 1}\ \underline{z}_o$ has been replaced by its proper magnitude $(k_2 = k_1^{\,2} + k_1^{\,2}(1-\eta^2)]^{1/2})$ and approximate direction.

For sufficiently large volumes (this time compared to the wavelength) and for slight to moderate losses $F(\underline{\hat{\varkappa}})$ is sharply peaked at $\underline{\hat{\varkappa}} \approx 0$ and Eq. (48) reduces to

$$< I(t)\, I^*(t-\tau) > \approx \frac{2}{\pi} \exp[jk_2 \rho_o \cdot \underline{v}\,\tau][\int d^3 \hat{\varkappa}\, F(\underline{\hat{\varkappa}}) e^{j\underline{\hat{\varkappa}} \cdot \underline{\bar{r}}_T}] M(0 \mid \tau) \tag{52}$$

where the bracketed term is readily evaluated as

$$\int d^3 \hat{\varkappa} F(\underline{\hat{\varkappa}}) e^{j\underline{\hat{\varkappa}} \cdot \underline{\bar{r}}_T} = (2\pi)^3 \int_V d^3 R \mid \gamma(Z) \mid^2 \exp[\frac{2\alpha Z}{\sqrt{1-\eta^{-2}}}] \delta(\underline{R} - \underline{\bar{r}}_T) =$$

$$= (2\pi)^3 |\gamma(z_T)|^2 \exp[\frac{2\alpha z_T}{\sqrt{1-\eta^{-2}}}] \tag{53}$$

The substitution of Eq. (53) into (52) and subsequently into Eq. (38), yields

$$C_E(\tau) \approx \frac{1 + 4\omega_o^{\,4} \mu_o^{\,2} M(0 \mid \tau)}{1 + 4\omega_o^{\,4} \mu_o^{\,2} M(0 \mid \tau = 0)} \exp[2jk_1 \rho_o \cdot \underline{v}\,\tau] \tag{54}$$

with

$$M(0\,|\tau) = \int d^3\varkappa \; \frac{\phi_\varepsilon(\varkappa\,|\tau)\,\sin^4\chi\,(k_2\underline{\rho_o}-\underline{\varkappa})\,\exp[-j\underline{\varkappa}\cdot\underline{v}\,\tau]}{[\,|k_2\underline{\rho_o}-\underline{\varkappa}\,|^2-\hat{k}_2^{\,2}]\,[\,|k_2\underline{\rho_o}-\underline{\varkappa}\,|^2-\hat{k}_2^{*2}]} \tag{55}$$

$\underline{v}$ is presumed horizontal as has been previously stated. It may be observed that since $\phi_\varepsilon(\varkappa\,|\tau)$ is symmetrical in τ it follows that $M(0\,|\,\tau) = M(0\,|-\tau)$ and the anticipated constraint $C_E(\tau) = C^*_E(-\tau)$ is satisfied. It is furthermore observed that upon the cancelation of the factor $|\gamma\,(z_T)\,|^2$ in Eq.(53) with that appearing in Eq.(38), ground effects totally disappear to the cited order of approximation. The lack of spectral sensitivity to the presence of the ground-vegetation interface is traceable to the fact that the target's motion is horizontal (parallel to the interface). Deviation from this configuration may result in a substantial increase of sensitivity to ground effects.

We now attempt the evaluation of the integral in (55) subject to the simplifying assumptions:

(a) $\underline{v} = v\underline{\rho_o}$ (motion toward or away from the receiver).

(b) $\phi_\varepsilon(\underline{\varkappa}\,|\tau) = \phi_\varepsilon(\varkappa\,|\tau)$, $\varkappa = |\underline{\varkappa}\,|$ (statistical isotropy).

The integration process is carried in the spherical coordinate system depicted in Fig.4. One has,

$$|k_2\underline{\rho_o} - \underline{\varkappa}\,|^2 = k_2^{\,2} + \varkappa^2 - 2k_2\varkappa\mu, \quad \mu = \cos\theta \tag{56}$$

or

$$|k_2\underline{\rho_o} - \underline{\varkappa}\,|^2 - \hat{k}_2^{\,2} = (\varkappa - \varkappa_1)\,(\varkappa - \varkappa_2) \tag{57}$$

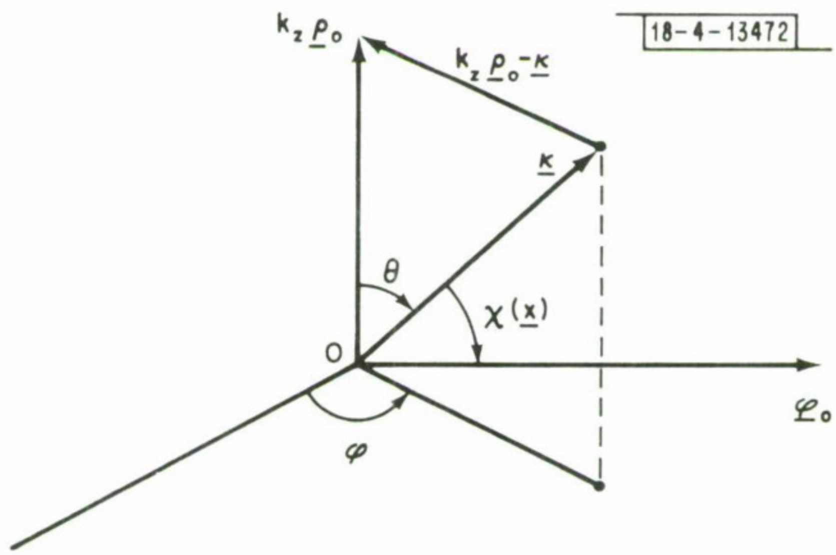

Fig. 4. Spherical coordinate system in wavenumber space.

where

$$\varkappa_{\substack{1\\2}} = k_2 \mu \left[1 \mp \sqrt{1 - \frac{2j\alpha}{k_2 \mu^2}} \right] \tag{58}$$

so that (55) becomes

$$M(0|\tau) = \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu \int_0^{\infty} d\varkappa \varkappa^2 \frac{\phi_\epsilon(\varkappa|\tau) \sin^4 \chi(\mu,\varphi) \exp[-j\varkappa v \mu \tau]}{(\varkappa - \varkappa_1)(\varkappa - \varkappa_2)(\varkappa - \varkappa_1^*)(\varkappa - \varkappa_2^*)} \tag{59}$$

The integrand in (59) has the property $f(\varkappa, \mu) = f(-\varkappa, -\mu)$ from which it follows that

$$\int_1^{-1} d\mu \int_0^{\infty} d\varkappa\, f(\varkappa,\mu) = \int_0^1 d\mu \int_{\infty}^{-\infty} d\varkappa\, f(\varkappa,\mu)$$

The limits of integration in Eq. (59) may be changed accordingly and the $\varkappa$-integration is readily performed by a residue calculation. The corresponding deformation of the integration path is depicted in Fig. 5 and leads to the result

$$M(0|\tau) = 2\pi j \int_0^{2\pi} d\varphi \int_0^1 d\mu \sin^4 \chi(\mu,\varphi) \left[\frac{\varkappa_1^{*2} \phi_\epsilon(\varkappa_1^*|\tau) \exp[-j\varkappa_1^* v\mu\tau]}{(\varkappa_1^* - \varkappa_1)(\varkappa_1^* - \varkappa_2)(\varkappa_1^* - \varkappa_2^*)} + \frac{\varkappa_2^2 \phi_\epsilon(\varkappa_2|\tau) \exp[-j\varkappa_2 v\mu\tau]}{(\varkappa_2 - \varkappa_1)(\varkappa_2 - \varkappa_1^*)(\varkappa_2 - \varkappa_2^*)} \right] \tag{60}$$

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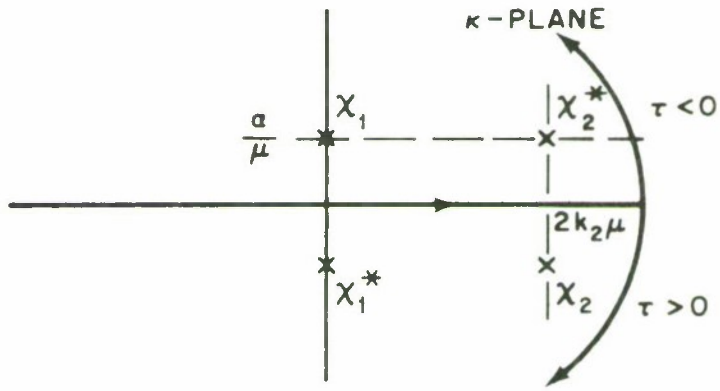
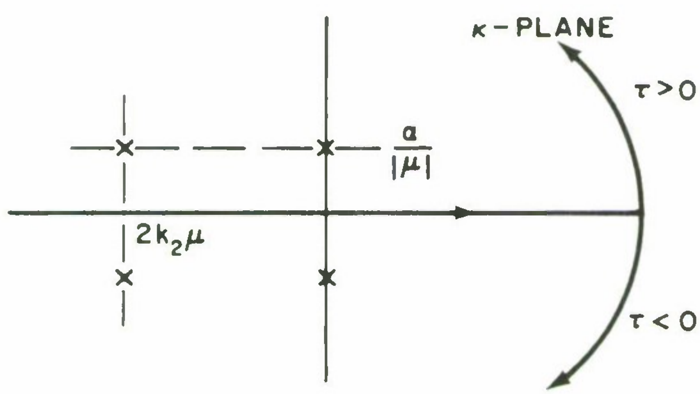


(a) $\mu > 0$



(b) $\mu < 0$

Fig. 5. The κ-plane description.

if $\tau > 0$ and to

$$M(0|\tau) = 2\pi j \int_0^{2\pi} d\varphi \int_0^1 d\mu \sin^4 \chi(\mu,\varphi) \left[\frac{\varkappa_1^2 \phi_\epsilon(\varkappa_1|\tau) \exp[-j\varkappa_1 v\mu\tau]}{(\varkappa_1-\varkappa_2)(\varkappa_1-\varkappa_1^*)(\varkappa_1-\varkappa_2^*)} + \right.$$

$$\left. + \frac{\varkappa_2^{*2} \varphi_\epsilon(\varkappa_2^*|\tau) \exp[-j\varkappa_2^* v\mu\tau]}{(\varkappa_2^*-\varkappa_1)(\varkappa_2^*-\varkappa_2)(\varkappa_2^*-\varkappa_1^*)} \right] \tag{61}$$

if $\tau < 0$.

The substitution of the explicit forms of $\varkappa_{1,2}$ given in Eq. (50) leads to rather cumbersome expressions. Although the analysis can be pursued further with these expressions we concentrate on the limit of small but nonvanishing loss (per wavelength) in which the following simplified forms

$$\varkappa_1 \approx j\frac{\alpha}{\mu}, \quad \varkappa_2 = 2k_2\mu - \frac{j\alpha}{\mu} \tag{62}$$

are obtained retaining first order terms in $\frac{\alpha}{\mu}$. Obviously the expansion of $\varkappa_{1,2}$ is invalid near $\mu \approx 0$ where the exact forms should be used. However, this corresponds to a small fraction of the range $0 < \mu \leq 1$ which, in turn, is associated with a small contribution to the ensuing integral. The contributions associated with the residues at $\varkappa_1^*$ and $\varkappa_1$ (first terms in Eqs. (60) and (61) respectively) are proportional to α and may be neglected. The only φ-dependent factor in Eqs. (60, 61) is (see Fig. 4).

$$\sin^4\chi(\mu,\varphi) = [1-\sin^2\varphi \sin^2\theta]^2 = [1-(1-\mu^2)\sin^2\varphi]^2 \tag{63}$$

which is readily integrated over φ, yielding

$$\beta(\mu) = \int_0^{2\pi} \sin^4\chi(\mu,\varphi)\, d\varphi = \int_0^{2\pi} [1-(1-\mu^2)\sin^2\varphi]^2\, d\varphi = \frac{\pi}{4}[3+2\mu^2+3\mu^4] \tag{64}$$

Retaining loss effects to lowest order and ignoring regions near $\mu \approx 0$ so that

$$\alpha << 2k_2\mu^2 \quad \text{and} \quad \alpha << (v\tau_c)^{-1}$$

one obtains,

$$M(0\,|\tau) = \frac{\pi}{\alpha} \int_0^1 d\mu\, \mu\, \beta(\mu)\, \phi_\epsilon(2k_2\mu\,|\tau)\, e^{-j2k_2\mu^2 v\tau} \tag{65}$$

We observe that Eq. (65) is not explicitly dependent on the scattering volume (V) as would be the case in a lossless case. This is indicative of the fact that the effective scattering volume is basically determined by the losses (Appendix A). The last observation does not contradict the low loss assumption introduced previously. α may indeed be small per wavelength or compared to $(v\,\tau_c)^{-1}$ and at the same time be sufficiently large to effectively restrict the scattering volume (V_g in Fig. A. 1 is typically very large).

Let us, for the time being, ignore the motion of the scattering centers represented by the τ dependence of ϕ_ϵ. We return to study this effect in Section E, below.

Let $u = \mu^2$ denote a new integration variable. Eq. (65) may be cast into the form

$$M(0\,|\tau) = \frac{\pi}{2\alpha} \int_0^1 du\, \beta(\sqrt{u})\, \phi_\epsilon(2k_2\sqrt{u})\, \exp[-2jk_2 uv\tau] \tag{66}$$

and the Fourier inversion of Eq. (54) results in

$$\phi_E^{(o)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_E(\tau)\, e^{-j\omega\tau}\, d\tau =$$

$$= [1 + 4\omega_o^4 \mu_o^2 M(0|\tau=0)]^{-1}\Big\{\delta(\omega-2k_1 v) + 4\omega_o^4 \mu_o^2 \frac{\pi}{2\alpha} \int_0^1 du\, \beta(\sqrt{u})\phi_\epsilon(2k_2\sqrt{u})$$

$$\delta(\omega-2k_1 v + 2k_2 vu)\Big\} \qquad (67)$$

or

$$\phi_E^{(o)}(\omega) = [1+4\omega_o^4 \mu_o^2 \ M(0|\tau = 0)]^{-1}\Big\{\delta(\omega - 2k_1 v) + \frac{\pi\omega_o^4 \mu_o^2}{\alpha k_2 v}\ \beta\Big(\sqrt{1-\frac{\omega}{2k_2 v}}\Big)$$

$$\phi_\epsilon\Big(2k_2 \sqrt{1- \frac{\omega}{2k_2 v}}\Big)\Big\} \qquad (68)$$

for $0 < \omega < 2k_2 v$ and equals to zero in the complementary part of the spectrum. The superscript (o), above indicates the fact that the motion of the scatters has been omitted. We terminate this section on the following notes:

(a) The singular term $\delta(\omega-2k_1 v)$, representing the direct target return, will be generally diffused owing to such factors as random or systematic variations of the target velocity, finite time broadening etc. It basically represents the target spectrum in the clear (i. e., in the total absence of multipath effects).

(b) The second term represents single scatter, multipath effects. It increases with decreasing α and disappears as α increases. It could be anticipated that the spectral return from dense, highly lossy vegetation slab will resemble that obtained in the clear.

(c) As α decreases, the relative significance of multipath effects increases to the point in which the effective scattering volume is no longer loss limited. This limit is irrelevant in the presently discussed physical context but is of great interest in most other situations.

(d) Eq. (68) constitutes a simple and readily interpretable result. The interpretation is presented in Section F. It facilitates the prediction of $\phi_E^{(o)}(\omega)$ provided that the spatial spectral density function (ϕ_ϵ) is known. Conversely, Eq. (68) may, under favorable circumstances, form a basis for the predictions of ϕ_ϵ from the measurement of the target spectrum ($\phi_E^{(o)}$) under no wind conditions.

E. EFFECTS DUE TO MOTION OF THE SCATTERING CENTERS.

The question of spectral perturbation caused by the random motion of the scatterers, characterized by a velocity field $\underline{U}$ (t), is discussed in Ref. 1 where the properties of the clutter return are investigated. The analysis is based on two major assumptions: (a) the random processes $\epsilon(\underline{r})$ (describing the spatial distribution of the scatterers) and $\underline{U}$ (t) are statistically independent and (b) $\epsilon(\underline{r}, t)$ is a conservative process, i.e.

$$\epsilon(\underline{r}, t + \tau) \ . \ \epsilon(\underline{r} - \int_t^{t+\tau} \underline{U}(s)\, ds,\ t) \qquad (69)$$

Consequently, the space-time correlation function defined in Eq. (40) is describable by

$$C_E(\underline{r}_1 - \underline{r}_2 \,|\tau) = < \epsilon(\underline{r}_1, t)\, \epsilon^*(\underline{r}_2 - \int_t^{t+\tau} \underline{U}(s)\, ds, t) > =$$

$$= < \hat{C}_\epsilon(\underline{r}_1 - \underline{r}_2 + \int_t^{t+\tau} \underline{U}(s)\, ds) >_u \tag{70}$$

where $\hat{C}_\epsilon$ is the spatial correlation and $< >_u$ denotes averaging over the velocity field. The spatial Fourier transformation of Eq. (70) results in,

$$\phi_\epsilon(\underline{\kappa}\,|\tau) = \frac{1}{(2\pi)^3} \int d^3\hat{r}\; e^{j\underline{\kappa}\cdot\hat{\underline{r}}} < \hat{C}_\epsilon(\underline{r} + \int_t^{t+\tau} \underline{U}(s)\, ds) >_u =$$

$$= \frac{1}{(2\pi)^3} \int d^3\hat{r}'\; e^{j\underline{\kappa}\cdot\hat{\underline{r}}'}\; \hat{C}_\epsilon(\hat{\underline{r}}') < \exp[-j\underline{\kappa} \cdot \int_t^{t+\tau} \underline{U}(s)\, ds] >_u$$

$$= \hat{\phi}_\epsilon(\underline{\kappa})\; w(\underline{\kappa}\,|\tau) \tag{71}$$

where

$$w(\underline{\kappa}\,|\tau) = < \exp[-j\underline{\kappa} \cdot \int_t^{t+\tau} \underline{U}(s)\, ds] >_u \tag{72}$$

, $\hat{\underline{r}} = \underline{r}_1 - \underline{r}_2$, $\hat{\underline{r}}' = \hat{\underline{r}} + \int_t^{t+\tau} \underline{U}(s)\, ds$ and $\hat{\phi}_\epsilon$ is the Fourier transform of $\hat{C}_\epsilon$ i.e. the spatial spectrum in the total absence of motion. The substitution of Eq. (71) into Eq. (55) leads to

$$M(0\,|\tau) = \int d^3\kappa \frac{\hat{\phi}_\epsilon(\kappa)\, \sin^4\chi\, w(\underline{\kappa}\,|\tau)\, \exp[-j\underline{\kappa} \cdot \underline{v}\,\tau]}{[\,|k_2\underline{\rho}_o - \underline{\kappa}|^2 - \hat{k}_2^{\,2}]\,[\,|k_2\underline{\rho}_o - \underline{\kappa}|^2 - \hat{k}_2^{*2}]} \tag{73}$$

If the random velocity field $\underline{U}(t)$ is isotropic it follows that w is exclusively a function of $\varkappa = |\underline{\varkappa}|$. It follows from Eq. (73), via a procedure analogous to that leading to Eq. (65), that

$$M(0|\tau) = \frac{\pi}{\alpha} \int_0^1 d\mu \, \mu \, \beta(\mu) \, \hat{\phi}_\epsilon(2k_2\mu) \, w(2k_2\mu|\tau) \exp[-j2k_2\mu^2 v\tau] \tag{74}$$

or analogously to Eq. (66)

$$M(0|\tau) = \frac{\pi}{2\alpha} \int_0^1 du \, \beta(\sqrt{u}) \, \hat{\phi}_\epsilon(2k_2\sqrt{u}|\tau) \, w(2k_2\sqrt{u}|\tau) \exp[-j2k_2 uv\tau] \tag{75}$$

The Fourier transformation of Eq. (58) becomes,

$$\phi_E(\omega) = [1 + 4\omega_o^4 \mu_o^2 M(0|\tau = 0)]^{-1} \Big\{ \delta(\omega - 2k_1 v) + 4\omega_o^4 \mu_o^2 \frac{\pi}{2\alpha} \int_o^1 du \, \beta(\sqrt{u})$$

$$\hat{\phi}_\epsilon(2k_2\sqrt{u}) \; W(2k_2\sqrt{u}|\omega - 2k_2 v + 2k_2 v u) \Big\} \tag{76}$$

where

$$W(2k_2\sqrt{u}|\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\tau \; w(2k_2\sqrt{u}|\tau) \exp[-j\omega\tau] \tag{77}$$

Eq. (77) reduces to (67) as $W(2k_2\sqrt{u}|\omega)$ approaches $\delta(\omega)$.

The nature of $W(2k_2\sqrt{u}|\omega)$ has been investigated in Ref. (1) where two alternative models have been suggested. The first of which involves the assumption that $\int_o^t \underline{U}(s)\,ds$ constitutes a multivariate Gaussian process possessing the features:

(a) $<\underline{U}> = 0$.

(b) Orthogonal velocity components are uncorrelated i.e. $<U_i U_j> = 0$, $i \neq j$.

(c) The velocity field is isotropic. Specifically, $<U_1^2> = <U_2^2> = <U_3^2> = \frac{1}{3}\sigma_u^2$.

One obtains, via Eq. (72),

$$w(\varkappa \mid \tau) = \exp\left[-\sigma_u^2 \varkappa^2 \int_0^\tau (\tau - s) C_u(s)\, ds \right] \tag{78}$$

where

$$C_u(s) = 3\sigma_u^{-2} < U_i(t)\, U_i(t+s) > \tag{79}$$

is the correlation coefficient which is independent of i owing to the presumed isotropy of $\underline{U}(t)$. Qualitative and quantitative details concerning the form (78) of $w(\varkappa \mid \tau)$ are presented in Ref. (1), so that further elaboration seems unnecessary. An alternative model (Ref. 1) views the velocity field as quasiharmonic, i. e.

$$\underline{U}(\underline{r}, t) = \Omega(\underline{r})\, \underline{\rho}_m(\underline{r}) \cos[\Omega(\underline{r})\, t + \varphi(\underline{r})] \tag{80}$$

where $\underline{\rho}_m$, Ω and φ are, assumedly, mutually independent random variables. The substitution of (80) in Eq. (72) and upon Fourier transforming the result one obtains

$$W(\underline{\varkappa} \mid \omega) = \sum_{n=-\infty}^{\infty} < J_n(\underline{\varkappa} \cdot \underline{\rho}_m)\, \delta(\omega - n\Omega) \exp[-jn\varphi + j\underline{\varkappa} \cdot \underline{\rho}_m \sin\varphi] >_{\varphi, \Omega, \underline{\rho}_m} \tag{81}$$

where the expansion

$$\exp[-j\underline{\varkappa} \cdot \underline{\rho}_m \sin(\Omega\tau + \varphi)] = \sum_{n=-\infty}^{\infty} J_n(\underline{\varkappa} \cdot \underline{\rho}_m)\, e^{-jn(\Omega\tau + \varphi)} \tag{82}$$

has been utilized and $< \; >_{\varphi, \Omega, \underline{\rho}_m}$ denotes sequential averaging over the random variables φ, Ω and $\underline{\rho}_m$, respectively. Assume the φ is uniformly distributed in the interval $(-\pi, \pi)$ and let $P(\Omega)$ denote the density function corresponding to Ω. Furthermore, assume $|\rho_{m\varkappa}|$ (the maximum value of the

projection of $\underline{\rho}$ along $\underline{\varkappa}$) to be Rayleigh distributed with $\hat{\rho}_m$ the locations of its peak. One obtains (see Ref. 1)

$$\exp[\varkappa^2 \hat{\rho}_m^2] W(\underline{\varkappa} | \omega) = I_o(\varkappa^2 \hat{\rho}_m^2) \delta(\omega) + \sum_{n=1}^{\infty} I_n(\varkappa^2 \hat{\rho}_m^2) [P(\frac{\omega}{n}) + P(-\frac{\omega}{n})] \qquad (83)$$

where $I_n(x)$ are the modified Bessel functions of the first kind. The following functional dependence for $P(\Omega)$ has been found suitable,

$$P(\Omega) \alpha \frac{(\Omega/\Omega_o)^p}{1 + b(\Omega/\Omega_o)^q} \qquad (84)$$

where Ω_o, b, p and q may be determined experimentally from the spectrum of clutter return at low winds.

F. THE TARGET SPECTRUM: HEURISTIC CONSTRUCTION.

The Doppler shift associated with the scattering events depicted in Fig. 6 are, neglecting relativistic effects, given by

$$\omega = k v (\cos \beta - \cos \alpha) \qquad (85)$$

where ω, k, and v denote, respectively, the Doppler shift, the wavenumber of the incident radiation, the target velocity (presumed constant). α and β are the angles between $\underline{v}$ and the wavevectors characterizing the waves incident upon and scattered by the target, respectively. Fig. 6a depicts a configuration in which the wave undergoes a random scattering event having been scattered by the target, while Fig. 6b describes the converse situation in which the random event occurs prior to the target scattering. The two cases result in identical shifts given by

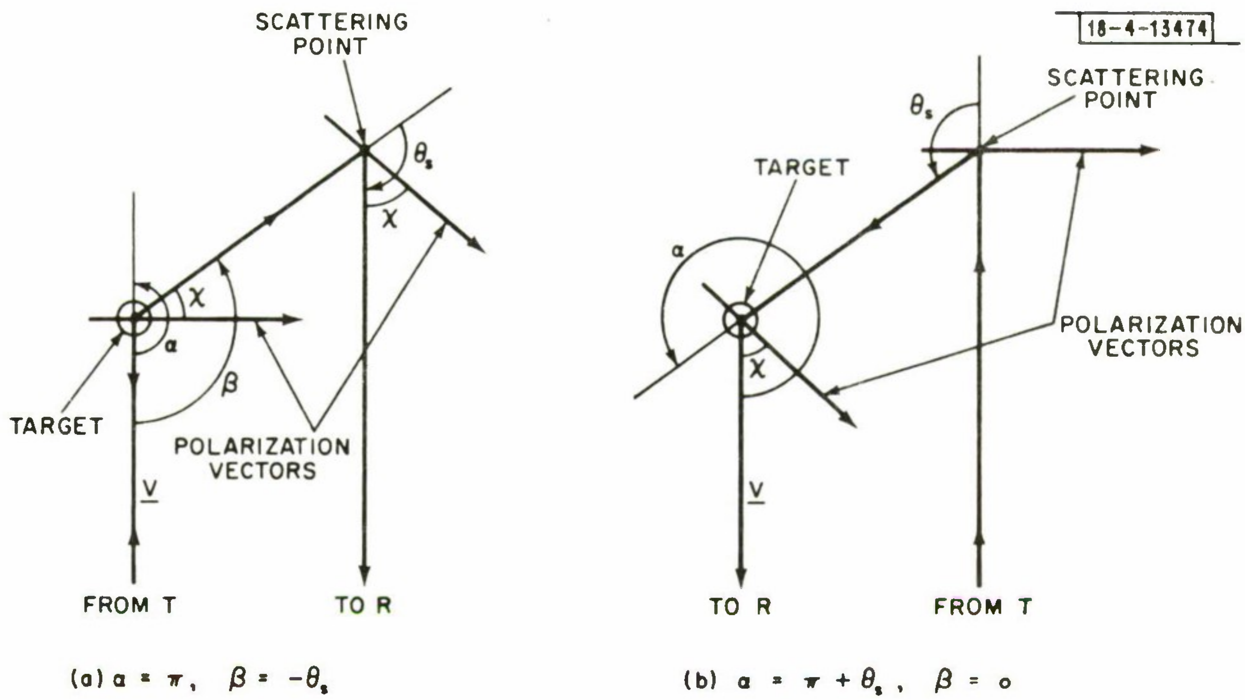


Fig. 6. Doppler shift by scattering.

$$\omega = kv[1 + \cos\theta_s] \tag{86}$$

where θ_s, the scattering angle, equals $-\beta$ in Fig. 6a and to $\alpha - \pi$ in Fig. 6b. It is noted that ω is never negative. Reversal of the shift direction is obtained only by the reversal of v. This is a direct consequence of the fact that $\underline{v}$ is presumedly directed toward the transmitter (receiver). The spectrum of the target return is determined by the mean power return from the entire range of the scattering angle θ_s. Consider a scattering volume on which a plane is incident. The effective scattering cross section of the volume is proportional to (2)

$$\sin^2\chi\, \hat{\Phi}_\epsilon(2k \sin\frac{\theta_s}{2}) \tag{87}$$

subject to the following constraints; (a) single scatter; (b) linear dimensions of V large compared to the correlation distance characterizing $\epsilon(\underline{r})$ in V; (c) $\epsilon(\underline{r})$ is statistically homogeneous; (d) $\epsilon(\underline{r})$ is statistically isotropic; (e) "far field" interaction. An additional "polarization" factor $\sin^2\chi$ appears due to the target scattering itself and (87) becomes

$$\sin^4\chi\, \hat{\Phi}_\epsilon(2k \sin\frac{\theta_s}{2}) \tag{88}$$

The angle χ is not exclusive function of θ_s. For a fixed value of θ_s, the factor $\sin^4\chi$ may be "averaged" over the angle φ depicted in Fig. 4, since variations in φ (with θ_s constant) do not correspond to spectral variations. The result of this averaging is the function $\beta/2\pi$ defined in Eq. (64). The substitution of (from (86))

$$\sin\frac{\theta_s}{2} = \sqrt{1 - \frac{\omega}{2kv}} \tag{89}$$

leads to the expression

$$\beta\left(1 - \frac{\omega}{2kv}\right) \hat{\phi}_{\epsilon}\left(2k \sqrt{1 - \frac{\omega}{2kv}}\right) \tag{90}$$

which is identical (disregarding proportionality constants) to the "multipath" term appearing in Eq. (68).

G. THE MEAN POWER RETURN

While the main thrust has been directed toward a gain of insight into the mechanisms which control the spectral characteristics of the target return an estimate of the mean power return is of great interest as well. The desired results stem directly from the non-normalized forms (35-36) evaluated at $\tau = 0$. For a matched antenna,

$$\langle P_T \rangle = \frac{\lambda^2}{4\pi} g(\varphi) \langle S \rangle = \frac{\lambda^2}{4\pi} g(\varphi) \tfrac{1}{2} \sqrt{\frac{\epsilon_o}{\mu_o}} \langle |\underline{E}_T|^2 \rangle =$$

$$= P_o \frac{g^2(\varphi)}{(2\pi)^2} \frac{\mu_o}{\epsilon_o} |\beta|^2 \frac{z^4 |\gamma(z_T)|^4}{(\eta^2 - 1)^2 \rho_o^8} \exp\left[\frac{4\alpha z_T}{\sqrt{1 - \eta^{-2}}}\right] \left[1 + 4\omega_o^4 \mu_o^2 M(0|0)\right] \tag{91}$$

where P_o, $g(\varphi)$ and $\langle S \rangle$ denote the transmitted power, the antenna gain at $\theta \approx 0$ and the Poynting vector at the receiver, respectively. $M(0|0)$ and $\gamma(z_T)$ are given in Eqs. (66) and (22), respectively, and $\beta(\sqrt{u})$ in (66) should not be confused with the constant polarizability factor introduced in Eq. (1) and appearing in Eq. (91), above. The first and second terms in (91) are recognized, respectively, as the contributions associated with the direct and multipath returns. The sensitivity of the mean return to medium

(scattering and absorption) losses (represented by the factor $\exp[\frac{4\alpha z_T}{\sqrt{1-\eta^{-2}}}]$), to ground proximity (represented by $|\gamma(z_T)|^2$), to range (represented by ρ_o^{-8}) and to antenna height above the fluctuating slab (represented by the factor z^4) are self-evident.

The signal to clutter power ratio is determined via a comparison of Eq. (91) with I(127)* resulting in

$$\langle P_T\rangle/\langle P_{clutter}\rangle = \frac{|\beta|^2 g^2(\varphi)\,|\gamma(z_T)|^4 \exp[2\alpha(2z_T+h)]\,[1+4\omega_o^4\mu_o^2 M|0|0)]}{\omega_o^2\rho_o\, dh\,[\int_{-\pi}^{\pi} g^2(\varphi)\,d\varphi]\,[\sinh(\frac{2\alpha h}{\sqrt{1-\eta^{-2}}})/\frac{2\alpha h}{\sqrt{1-\eta^{-2}}}]\,\phi_\varepsilon(2k_1, 2k_1\sqrt{\eta^2-1})} \tag{92}$$

In (92) h and d denote the effective forest-slab height and the depth of the range cell, respectively. The first and second arguments in ϕ_ε, above, refer to the respective horizontal ($|\underline{\kappa}_t| = |\underline{\kappa} - \underline{z}_o\kappa_z| = 2k_1$) and vertical ($\kappa_z = 2k_1\sqrt{\eta^2-1}$) components of $\underline{\kappa}$.

*Eq. I (127) contains two typographical errors: (a) the factor (η^2-1) in the denominator should be squared and (b) the sin h argument should read $2\alpha h/\sqrt{1-\eta^{-2}}$.

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APPENDIX A: THE EFFECTIVE SCATTERING VOLUME

The volume which actively participates in the scattering process described in the text is defined by the intersection of three factors: (a) the finite extent of the slab, (b) the radar gating and (c) the background (effective) loss. (a) above requires no further comment. The effect of the timing constraints (b) is depicted in Fig. A. 1 in which ground effects are omitted (but could be readily accounted for by appropriate imaging). The scattering volume (V_g), defined consistently with the radar gating, constitutes a paraboloid of revolution (with z'' the axis of symmetry) determined below. Consider the cross section of the scattering volume with the plane $y'' = 0$ (Fig. A. 1). The phase accumulated by the wave incident on the target having undergone a scattering event at the point (x'', z'') is a function of (x'', z'') given by

$$k_2[\sqrt{x''^2 + z''^2} - z''] = \text{constant} \tag{A.1}$$

stemming from the fact that the incident phase front (prior to scattering) or the phase front of the backscattered signal (having undergone a scattering event) are approximately planar (i. e. θ_2 is a constant angle, insensitive to variations in θ_1). Eq. (A. 1) yields the family of parabolas given by

$$x''^2 - 2cz'' = c^2, \; c > 0 \tag{A.2}$$

or in the x' - z' coordinate system:

$$x'' = x' \cos\theta_2 + z' \sin\theta_2, \; z'' = z' \cos\theta_2 - x' \sin\theta_2 \tag{A.3}$$

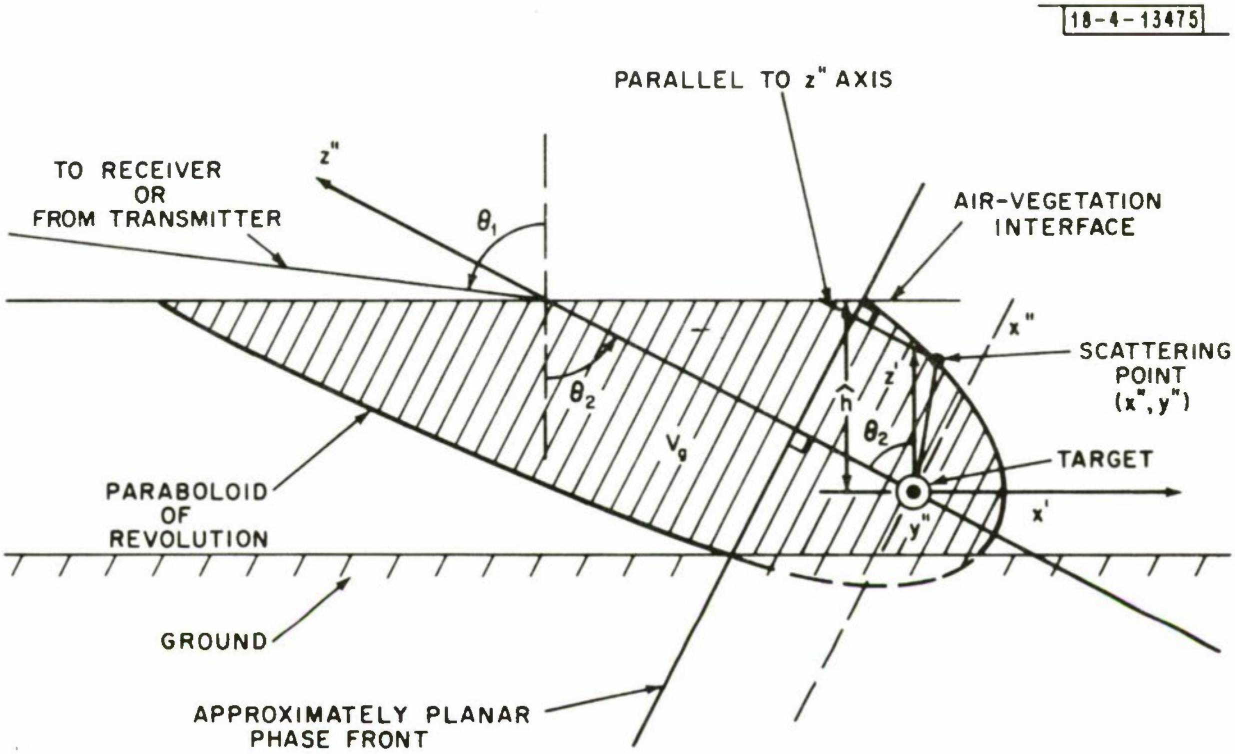

Fig. A.1. Gating limited scattering volume.

one has

$$(\sqrt{1-\eta^{-2}}\ x' + \frac{z'}{\eta})^2 - 2c(\sqrt{1-\eta^{-2}}\ z' - \frac{x'}{\eta}) = c^2 \tag{A.4}$$

in which the approximate form $\sin\theta_2 \approx \eta^{-1}$ has been utilized. The maximum value that c may get is determined by the size of the range cell. It is not hard to show that all scattering points external to the paraboloid characterized by $c > c_{max} = \frac{d}{2}$ (d - the range cell depth) will not reach the receiver on time.

The third constraint on the scattering volume, the background loss, is probably the most important under the prevailing circumstances. The loss limited volume may be defined as the locus of all scattering points (x', y', z') obeying the inequality

$$\alpha[-L + L' + L''] \leq N \tag{A.5}$$

where N is a constant representing the maximum loss difference between the paths $L'' + L'$ and L (Fig. A.2) beyond which we may totally neglect the scattered wave contribution. α is the loss rate defined in the text and the geometrical length segments are shown in Fig. (A.2). The volume bounds are given by

$$\frac{\hat{h} - z'}{\cos\theta_2} + [\rho'^2 + z'^2]^{\frac{1}{2}} - \frac{\hat{h}}{\cos\theta_2} = \frac{N}{\alpha} \tag{A.6}$$

or

$$\rho'^2 = (\frac{N}{\alpha} + \frac{z'}{\cos\theta_2})^2 - z'^2 = (\frac{N}{\alpha} + \frac{z'}{\cos\theta_2} - z')(\frac{N}{\alpha} + \frac{z'}{\cos\theta_2} + z') \tag{A.7}$$

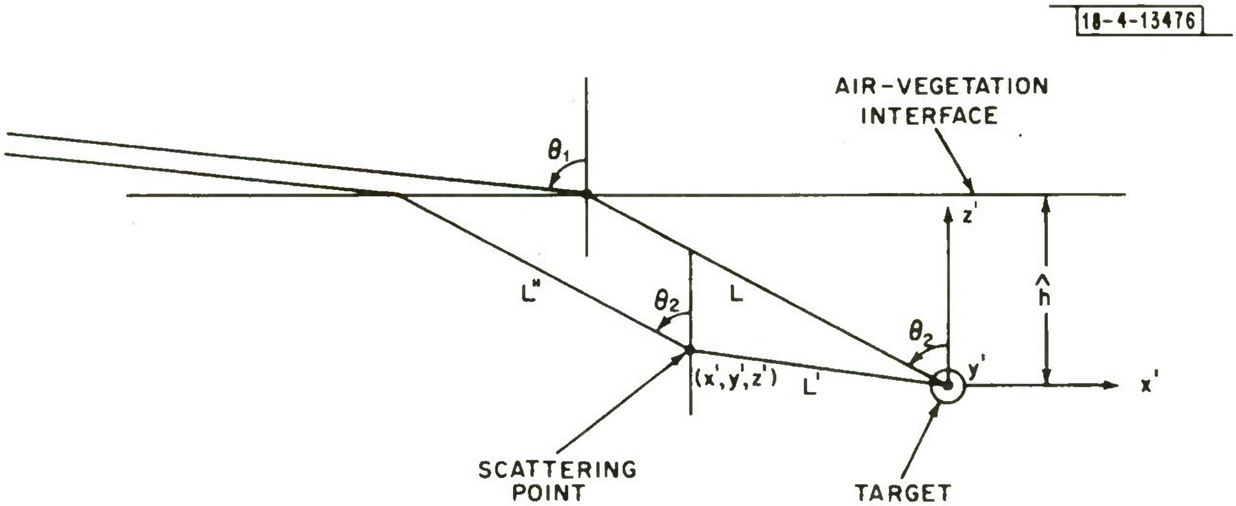

Fig. A.2. Loss constraints.

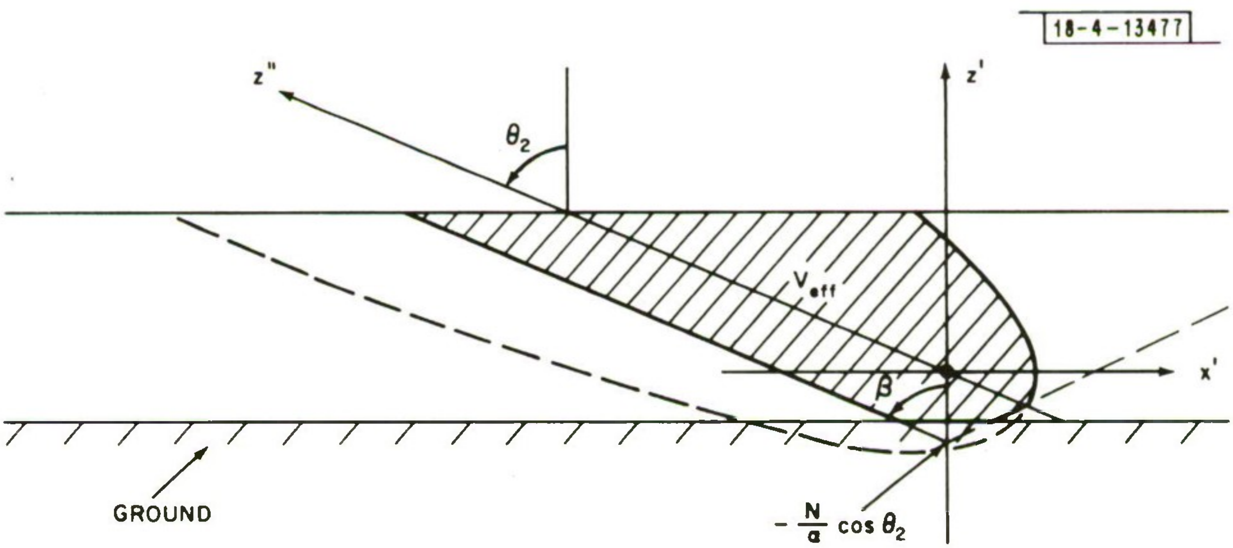

Fig. A.3. The effective scattering volume.

Approximately, presuming $\cos\theta_2 = [1-\eta^{-2}]^{\frac{1}{2}} << 1$ one has

$$\rho' = \pm \left(\frac{N}{\alpha} + \frac{z'}{\cos\theta_2} \right) \tag{A.8}$$

The effective scattering volume is depicted in Fig. (A.3) in which the angle β is given by

$$\tan\beta = \frac{1}{\cos\theta_2} \approx [1-\eta^{-2}]^{-\frac{1}{2}} \tag{A.9}$$

and from $|\eta - 1| << 1$ it follows that $\beta \approx \theta_2$. In Fig. (A.3), we are reminded again, that ground effects have been ignored but may be readily accounted for by suitable imaging and an analogous construction sequence. This extension is generally important for ground targets but is insignificant for spectral considerations as long as the target motion is horizontal.

APPENDIX B: THE DETERMINATION OF EQ. (27)

$$\underline{p} \equiv (\underset{\sim}{1} - \frac{(\underline{r}-\underline{r}_1)(\underline{r}-\underline{r}_1)}{|\underline{r}-\underline{r}_1|^2}) \cdot (\underset{\sim}{1} - \frac{(\underline{r}_1-\underline{r}_T)(\underline{r}_T-\underline{r}_T)}{|\underline{r}_1-\underline{r}_T|^2} \cdot \underline{\varphi}_o = \tag{B. 1}$$

$$= \underline{\varphi}_o - \frac{(\underline{r}-\underline{r}_1)(\underline{r}-\underline{r}_1)\cdot\underline{\varphi}_o}{|\underline{r}-\underline{r}_1|^2} - \frac{(\underline{r}_1-\underline{r}_T)}{|\underline{r}_1-\underline{r}_T|}\cos\chi + \frac{(\underline{r}-\underline{r}_1)(\underline{r}-\underline{r}_1)\cdot(\underline{r}_1-\underline{r}_T)}{|\underline{r}-\underline{r}_1|^2\,|\underline{r}_1-\underline{r}_T|}\cos\chi$$

where $\cos\chi = \underline{\varphi}_o \cdot (\underline{r}_1-\underline{r}_T)/|\underline{r}_1-\underline{r}_T|$.

Under the assumption $\bar{\underline{r}} >> |\underline{r}_1-\underline{r}_T|$ we have $(\underline{r}-\underline{r}_1)\cdot\underline{\varphi}_o \approx 0$ and $\frac{\underline{r}-\underline{r}_1}{|\underline{r}-\underline{r}_1|} \approx \underline{\rho}_o$

hence,

$$\underline{p} \approx \underline{\varphi}_o - [\underset{\sim}{1} - \frac{(\underline{r}-\underline{r}_1)(\underline{r}-\underline{r}_1)}{|\underline{r}-\underline{r}_1|^2}] \cdot \frac{\underline{r}_1-\underline{r}_T}{|\underline{r}-\underline{r}_T|}\cos\chi \approx \underline{\varphi}_o - [\underset{\sim}{1} - \underline{\rho}_o\underline{\rho}_o] \cdot \frac{\underline{r}_1-\underline{r}_T}{|\underline{r}_1-\underline{r}_T|}\cos\chi$$

$$\approx \underline{\varphi}_o - [\underline{z}_o\,\underline{z}_o + \underline{\varphi}_o\underline{\varphi}_o] \cdot \frac{\underline{r}_1-\underline{r}_T}{|\underline{r}_1-\underline{r}_T|}\cos\chi \tag{B. 2}$$

Ignoring the vertical component (not sensed by the receiving antenna) results in

$$\underline{p} \approx \underline{\varphi}_o \sin^2\chi \tag{B. 3}$$

Similarly, under the assumptions above,

$$\hat{\underline{p}} \equiv (\underset{\sim}{1} - \frac{(\underline{r}-\underline{r}_T)(\underline{r}-\underline{r}_r)}{|\underline{r}-\underline{r}_T|^2}) \cdot (\underset{\sim}{1} - \frac{(\underline{r}_1-\underline{r}_T)(\underline{r}_1-\underline{r}_T)}{|\underline{r}_1-\underline{r}_T|^2}) \cdot \underline{\varphi}_o \tag{B. 4}$$

since

$$\underset{\sim}{1} - \frac{(\underline{r} - \underline{r}_T)(\underline{r} - \underline{r}_T)}{|\underline{r} - \underline{r}_T|^2} \approx \underset{\sim}{1} - \underline{\rho}_o \underline{\rho}_o \qquad (B.5)$$

we have,

$$\hat{\underline{p}} \approx \underline{\varphi}_o - (\underset{\sim}{1} - \underline{\rho}_o \underline{\rho}_o) \cdot \frac{\underline{r}_1 - \underline{r}_T}{|\underline{r}_1 - \underline{r}_T|} \cos\chi \approx \underline{p} \qquad (B.6)$$

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13. ABSTRACT

The stochastic nature of the target return depends on numerous factors: the spatial randomness of the forest's EM parameters, their temporal fluctuations, the target size and its state of motion. Our attention is focused upon the calculation of the mean power and the temporal spectrum of the target return, quantities of direct and obvious relevance. The spectral modifications are traceable to two first order factors: (1) the motion of the scattering centers within the random vegetation slab and (2) the target motion through a randomly inhomogeneous field, with the latter likely to dominate.

The analysis is carried out within the framework of the so-called "distorted wave Born approximation." The incoherent (random) scattering is accounted for to the order of single scatter, while coherent (background) effects such as refraction at the air-vegetation interface and ground reflections are properly considered. The rather laborious analytical sequence leads (via a set of carefully listed simplifying assumptions) to final results which are analytically simple, numerically tractable and their physical content readily interpretable. Two major assumptions concerning target characteristics should be mentioned: (1) the study dealt exclusively with point targets and (2) the target is presumed to move toward or away from the radar with constant velocity.

14. KEY WORDS

backscatter	vegetated environment
target return	point targets